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Effect of the Response Delay of the Measuring System on Thermal Diffusivity Measurement Using the Flash Method[1]

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This work investigates the effect of the response delay of a measuring system on a thermal diffusivity measurement. A model of an mth-order delay in the measuring system is introduced, and a general expression for the output of the system with temperature response as input is derived. The effect on the temperature response caused by such a system is discussed. As a practical example, a third-order measuring system is considered. The measured temperature responses of stainless steel foils are compared with those calculated with the model of a third-order delay system. Good agreement between the two results is shown.

KEY WORDS: flash method; response delay; thermal diffusivity; time constant.

1. INTRODUCTION

It has been suggested that the flash method [1] should be a good method not only for thick specimens, but also for thin foils. When the method is employed to measure thin specimens, in order to obtain the temperature responses correctly, the measuring system including devices such as a sensor, preamplifier, etc., should be sufficiently fast. A response delay of the devices may result in a significant measurement error. For a one-time-constant system, the error due to response delay has been discussed by Araki and Natsui [2] by introducing a time delay model as used in a low-pass

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filter. In practice, however, a system for measuring temperature response usually includes multiple devices, which cannot be described by one time constant.

In this paper, a measuring system with multiple devices is considered. A general expression for the output of the system with temperature response as input is derived. The effect on temperature response caused by such a system is discussed. As a practical example, we consider a measuring system used in our laboratory, which includes a (Hg, Cd)Te IR detector with a time constant of 0.9 μs, a preamplifier of 0.64 μs, and a filter of 0.115 μs in series. The measured temperature responses in stainless steel foils with thicknesses from 8 to 90 μm heated by a 15-ns YAG laser are compared with calculations.

2. MODEL OF MEASURING SYSTEM WITH TIME DELAY

A system with m devices arranged in series is usually called an mth-order system. According to Araki and Natsui [2], the delay of a system with one time constant can be described by a first-order delay equation. The time constant is determined by providing a stepwise input and measuring the response time of the output as shown in Fig. 1.

In the present study, an mth-order system as shown in Fig. 2 is considered in which U1, U2, U3,..., Um are the components of the measuring system, corresponding to electronic devices such as a sensor, amplifier, filter, etc., with time constants $\tau_1, \tau_2, \tau_3, \ldots, \tau_m$, respectively. If all of the devices in the system are assumed to have the same delay process as shown in Fig. 1, the delay between the input and output of the mth-order system can be described by the following set of equations:

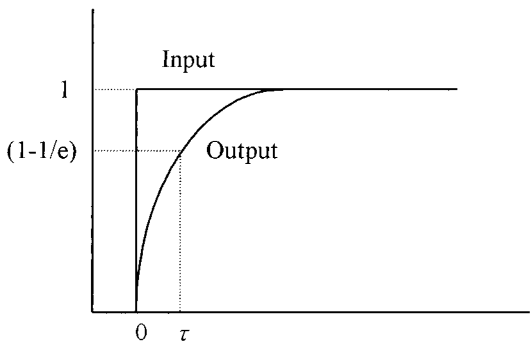

Fig. 1. Response time of a device in a measuring system.

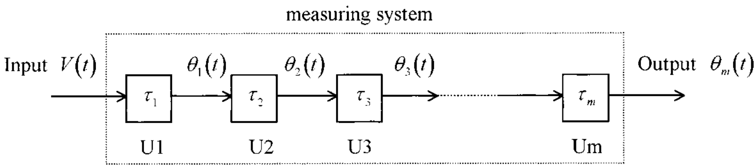

Fig. 2. Block diagram of an mth-order delay system.

$$\tau_1 \frac{d\theta_1(t)}{dt} + \theta_1(t) = V(t) \tag{1a}$$

$$\tau_i \frac{d\theta_i(t)}{dt} + \theta_i(t) = \theta_{i-1}(t), \qquad i = 2, 3,..., m \tag{1b}$$

which can be written in integral form as follows:

$$\theta_1(t) = e^{-t/\tau_1} \left[\int \frac{e^{t/\tau_1}}{\tau_1} V(t)\, dt + C_1 \right] \tag{2a}$$

$$\theta_i(t) = e^{-t/\tau_i} \left[\int \frac{e^{t/\tau_i}}{\tau_i} \theta_{i-1}(t)\, dt + C_i \right], \qquad i = 2, 3,..., m \tag{2b}$$

where C_i, $i = 1, 2, 3,..., m$, are constants determined by initial conditions.

3. TEMPERATURE RESPONSE AS OUTPUT OF AN mTH-ORDER DELAY SYSTEM

According to Parker et al. [1], the normalized temperature response at the near surface of a specimen with short, pulsewise heating at the front surface is

$$V(t) = 1 + 2 \sum_{n=1}^{\infty} (-1)^n e^{-n^2 t/t_0} \tag{3}$$

where $t_0 = L^2/(\pi^2 a)$. Substituting Eq. (3) into Eq. (2) and using the initial conditions

$$V(0) = \theta_i(0) = 0, \qquad i = 1, 2,..., m \tag{4}$$

we can obtain the temperature response as the output of the mth-order delay system. For $m = 1$,

$$\theta_1(t) = 1 - e^{-t/\tau_1} + 2 \sum_{n=1}^{\infty} (-1)^n \left[\frac{e^{-n^2 t/t_0}}{1 - n^2 \tau_1/t_0} - \frac{e^{-t/\tau_1}}{1 - n^2 \tau_1/t_0} \right] \tag{5}$$

For $m = 2, 3, \ldots$, the output of the system is derived for the following two cases.

1. Case where $\tau_1 \neq \tau_2 \neq \cdots \neq \tau_m$. By using Eqs. (5) and (2b), the temperature responses for $m = 2$ and 3 are obtained as

$$\theta_2(t) = 1 - \frac{\tau_1 e^{-t/\tau_1}}{\tau_1 - \tau_2} - \frac{\tau_2 e^{-t/\tau_2}}{\tau_2 - \tau_1} + 2\sum_{n=1}^{\infty} (-1)^n \left[\begin{array}{c} \dfrac{e^{-n^2 t/t_0}}{(1 - n^2\tau_1/t_0)(1 - n^2\tau_2/t_0)} \\ -\dfrac{\tau_1 e^{-t/\tau_1}}{(\tau_1 - \tau_2)(1 - n^2\tau_1/t_0)} \\ -\dfrac{\tau_2 e^{-t/\tau_2}}{(\tau_2 - \tau_1)(1 - n^2\tau_2/t_0)} \end{array} \right] \tag{6}$$

$$\begin{aligned} \theta_3(t) = & \left[\begin{array}{c} 1 - \dfrac{\tau_1^2 e^{-t/\tau_1}}{(\tau_1 - \tau_2)(\tau_1 - \tau_3)} \\ -\dfrac{\tau_2^2 e^{-t/\tau_2}}{(\tau_2 - \tau_1)(\tau_2 - \tau_3)} \\ -\dfrac{\tau_3^2 e^{-t/\tau_3}}{(\tau_3 - \tau_1)(\tau_3 - \tau_2)} \end{array} \right] \\ & + 2\sum_{n=1}^{\infty} (-1)^n \left[\begin{array}{c} \dfrac{e^{-n^2 t/t_0}}{(1 - n^2\tau_1/t_0)(1 - n^2\tau_2/t_0)(1 - n^2\tau_3/t_0)} \\ -\dfrac{\tau_1^2 e^{-t/\tau_1}}{(\tau_1 - \tau_2)(\tau_1 - \tau_3)(1 - n^2\tau_1/t_0)} \\ -\dfrac{\tau_2^2 e^{-t/\tau_2}}{(\tau_2 - \tau_1)(\tau_2 - \tau_3)(1 - n^2\tau_2/t_0)} \\ -\dfrac{\tau_3^2 e^{-t/\tau_3}}{(\tau_3 - \tau_1)(\tau_3 - \tau_2)(1 - n^2\tau_3/t_0)} \end{array} \right] \end{aligned} \tag{7}$$

From Eqs. (5)–(7), it is not difficult to write out the formula for the mth-order system as follows:

$$\begin{aligned} \theta_m(t) = & 1 - \sum_{i=1}^{m} \frac{\tau_i^{m-1} e^{-t/\tau_i}}{\prod_{j=1,\, j\neq i}^{m} (\tau_i - \tau_j)} + 2\sum_{n=1}^{\infty} (-1)^n \\ & \times \left[\frac{e^{-n^2 t/t_0}}{\prod_{i=1}^{m} (1 - n^2\tau_i/t_0)} - \sum_{i=1}^{m} \frac{\tau_i^{m-1} e^{-t/\tau_i}}{(1 - n^2\tau_i/t_0) \prod_{j=1,\, j\neq i}^{m} (\tau_i - \tau_j)} \right] \end{aligned} \tag{8}$$

Then, assuming Eq. (8) is valid as a general formula, according to Eq. (2b), the formula for the $(m+1)$th-order should be

$$\theta_{m+1}(t)=e^{-t/\tau_{m+1}}\left[\int \frac{e^{t/\tau_{m+1}}}{\tau_{m+1}}\,\theta_m(t)\,dt+C_{m+1}\right] \tag{9}$$

Substituting Eq. (8) into Eq. (9) and using the initial condition given by (4), we obtain, by a straightforward series of manipulations of integration, the temperature response output from an $(m+1)$th-order delay system:

$$\begin{aligned}\theta_{m+1}(t)=1-&\sum_{i=1}^{m+1}\frac{\tau_i^m e^{-t/\tau_i}}{\prod_{j=1,\,j\neq i}^{m}(\tau_i-\tau_j)}\\ &+2\sum_{n=1}^{\infty}(-1)^n\left[\frac{e^{-n^2t/t_0}}{\prod_{i=1}^{m+1}(1-n^2\tau_i/t_0)}-\sum_{i=1}^{m+1}\frac{\tau_i^m e^{-t/\tau_i}}{(1-n^2\tau_i/t_0)\prod_{j=1,\,j\neq i}^{m+1}(\tau_i-\tau_j)}\right]\end{aligned} \tag{10}$$

Comparing Eqs. (8) and (10), and according to mathematical induction, Eq. (8) is proved to be the general formula for an mth-order delay system.

2. Case where $\tau_1=\tau_2=\cdots=\tau_m=\tau$. The same method is used as in the previous case. Then we obtain the formula

$$\begin{aligned}\theta_m(t)=1-e^{-t/\tau}&\sum_{i=1}^{m}\frac{1}{(i-1)!}\left(\frac{t}{\tau}\right)^{i-1}\\ &+2\sum_{n=1}^{\infty}(-1)^n\left[\frac{e^{-n^2t/t_0}}{(1-n^2\tau/t_0)^m}-e^{-t/\tau}\sum_{i=1}^{m}\frac{1}{(i-1)!}\frac{1}{(1-n^2\tau/t_0)^{m-i+1}}\left(\frac{t}{\tau}\right)^{i-1}\right]\end{aligned} \tag{11}$$

4. EFFECT OF SYSTEM DELAY ON THE MEASUREMENT OF THERMAL DIFFUSIVITY

Utilizing Eqs. (10) and (11), we can show the effects of the delay on temperature responses and thermal diffusivities for any multi-order system. In the present study, a third-order system is considered because transient temperature-measuring equipment is typically a third-order system, including an IR detector, a preamplifier, and a filter.

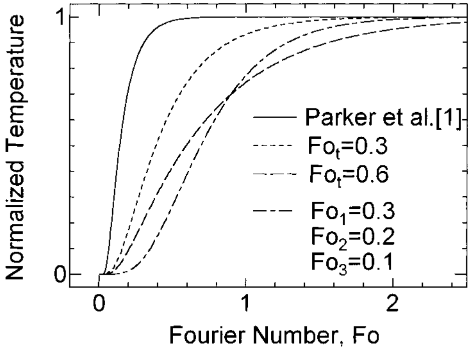

Fig. 3. Calculated temperature responses with and without delay considered.

Figure 3 shows a typical output of a third-order system with Parker's temperature response as input in dimensionless form. In this figure, the Fourier numbers are defined as

$$Fo = \frac{at}{L^2}, \qquad Fo_t = \frac{a\tau}{L^2}, \qquad Fo_i = \frac{a\tau_i}{L^2}$$

where the Fo_i $(i = 1, 2, 3)$ express the response delays of the three devices. Fo_t expresses the response delay of the system if it is treated as a first-order system. The comparison between the temperature responses with and without delay considered shows that the measured temperature rise becomes slower than the real one because of the response delay of the measuring system, and this will result in a higher predicted value of the half-time (time required to reach half the maximum value of temperature). The curve for $Fo_t = 0.3$ gives the temperature response as the output of a first-order system with a time constant that has the same value as the longest one in the third-order system. The curve for $Fo_t = 0.6$ gives a temperature response as the output of a first-order system with a time constant whose value is the sum of three devices in the third-order system. The differences between these curves and those of the third-order system show that the delay of a third-order system cannot be described simply by the longest time constant of the three devices or by the sum of the time constants.

As a practical example, we used the multi-order system model to analyze the data measured by laser flash equipment in our laboratory. This equipment includes a 15-ns YAG pulse laser as a heating source and a third-order measuring system for the temperature response made up of a (Hg, Cd)Te IR detector with a time constant of 0.9 μs, a preamplifier of

0.64 μs, and a filter of 0.115 μs arranged in series. We used this equipment to measure the temperature responses in stainless steel foils with thicknesses from 8 to 90 μm, and we calculated the thermal diffusivities from these temperature responses. The results are shown in Figs. 4 and 5.

In Fig. 4, the measured temperature responses for thicknesses of 50 and 10 μm are compared with calculated values using the formula for the third-order system and with Parker et al. [1]. For thicker specimens (for example, 50 μm), the measuring system is sufficiently fast for the temperature responses. The calculations without delay considered show good agreement with the experiments, while for thinner specimens (for example, 10 μm), the experimental temperature responses become slower than the calculated results using Parker's formula. However, for all specimens, the calculations for a third-order system show good agreement with experiments.

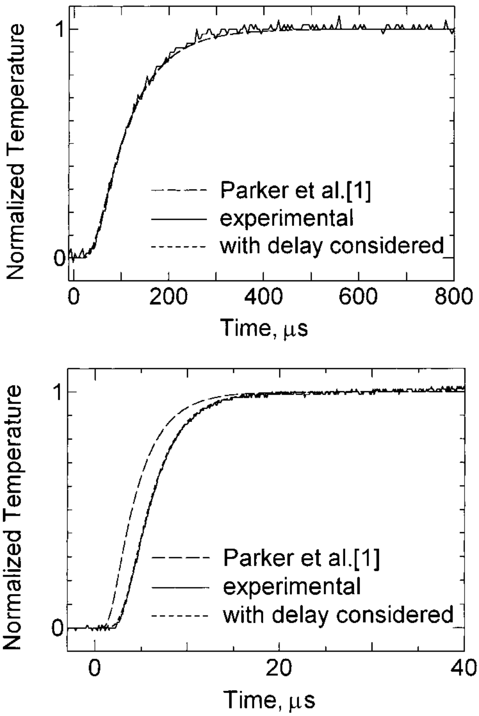

Fig. 4. Comparison of experimental and calculated temperature responses of SUS-304. (a) Thickness of 50 μm (the two calculated results overlap the experimental result). (b) Thickness of 10 μm (the result with delay considered overlaps the experimental result).

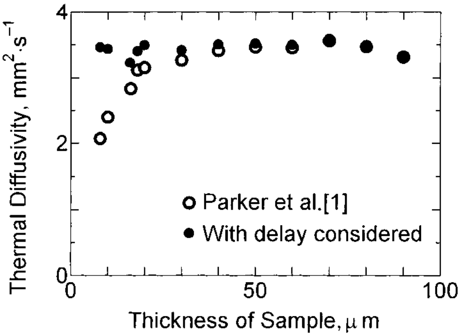

Fig. 5. Measured thermal diffusivities of SUS-304 for various thicknesses.

Figure 5 shows the comparison of thermal diffusivities calculated by Parker's half-time method and by fitting experimental data using the formula for a third-order system. It can be seen that the data using the formula with delay considered are almost independent of thickness, while those using the formula without delay considered show a sharp decrease when the specimens are thinner than 20 μm.

5. CONCLUSION

The general expression for the output of a measuring system with multi-order delay is derived. The temperature responses calculated with this formula show good agreement with experiments. This suggests that, if the time constants of all devices in a system are accurately known, it is possible to estimate the thermal diffusivity of thin specimens with temperature responses faster than the measuring system's response.

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